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A note on reverse transition

By P. BRADSHAW
Aerodynamics Division, National Physical Laboratory, Teddington

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It is shown that the criteria of Preston and of Patel & Head are special cases of a general Reynolds-number criterion which can be deduced as a requirement that there shall be some part of the boundary layer (in practice, the inner layer) in which the energy-containing and dissipating ranges of eddy size do not quite overlap.

1. Introduction

Reversion of turbulent to laminar flow, alias retransition, reverse transition or relaminarization, has been defined and explained in different ways by different authors. Preston (1958), whose argument was applied to the boundary layer in zero pressure gradient but should be more generally valid, suggested that turbulent flow could be maintained only if there was a perceptible inner law region between the outer edge of the viscous region (say $u_\tau y/\nu = 30$) and the inner edge of the outer layer (say $y/\delta = 0{\cdot}2$): the condition $0{\cdot}2u_\tau\delta/\nu > 30$ implies $U_1\delta_2/\nu > 320$ for a boundary layer in zero pressure gradient. Patel & Head (1968) have taken departure from the inner-law velocity profile (to be precise, an 'overshoot' of velocity above the universal logarithmic profile) as a criterion of the start of reverse transition in strong favourable pressure gradient, leading to a critical value of $\Delta_\tau \equiv (\nu/u_\tau^3)\,\partial\tau/\partial y$ of about $-0{\cdot}009$. Badri Narayanan & Ramjee (1968), using their own observations and those of others, have tentatively identified three states: (i) disappearance of the large eddy structure near the wall at a critical value of $(\nu/U_1^2)\,dU_1/dx$; (ii) departure from the inner law velocity profile at a critical value of $(\nu/u_\tau^3)\,dp/dx$ or $(\nu/u_\tau^3)\,\partial\tau/\partial y$; (iii) decay of turbulent intensity starting at a critical value of $U_1\delta_2/\nu$; experimental data are too scanty and too unreliable to make this distinction certain.

Launder & Stinchcombe (1967) and Patel & Head (1968) have traced a smooth progression from fully turbulent to fully laminar velocity profiles as the Reynolds number is decreased: clearly, reverse transition does not imply any sudden change in mean properties. At least in some cases the smooth progression reflects a smooth change in the relative probability of laminar or turbulent patches: evidently a small, well-organized spot or slug of turbulence can maintain itself at a lower Reynolds number than a large body of turbulence.

Although it is clear that measurements of mean properties in a part-laminar, part-turbulent flow must be treated with caution, the practical need is for a general 'Reynolds number' criterion, based on mean properties, to indicate the beginning of reverse transition or, what is in principle the same thing, the lowest Reynolds number at which fully turbulent flow can exist. The purpose of the present paper is to show that the criteria of Preston and of Patel & Head are special cases of such a general Reynolds number criterion with a simple (i.e. crude) physical interpretation.

2. Effect of viscosity on turbulent eddies

A turbulent flow will become directly dependent on viscosity when the energy-containing (or shear-stress-producing) and dissipating ranges of eddy size overlap. A typical length scale of the energy-containing eddies is

$$(\tau/\rho)^{\frac{3}{2}}/(\text{rate of energy transfer from energy-containing eddies})$$

but since the denominator is necessarily equal to the dissipation rate ϵ, the length scale is equal to the dissipation length parameter $L \equiv (\tau/\rho)^{\frac{3}{2}}/\epsilon$. The Kolmogorov length scale of the dissipating eddies is $(\nu^3/\epsilon)^{\frac{1}{4}} = L/\{(\tau/\rho)^{\frac{1}{2}}L/\nu\}^{\frac{3}{4}}$ so that the ratio of the two scales is $\{(\tau/\rho)^{\frac{1}{2}}L/\nu\}^{\frac{3}{4}}$ and appreciable overlap will occur below some critical value of the 'eddy Reynolds number' $(\tau/\rho)^{\frac{1}{2}}L/\nu$. It so happens that, if the production rate $(\tau/\rho)\,\partial U/\partial y$ is equal to the dissipation rate ϵ so that L becomes equal to the apparent mixing length, the eddy Reynolds number is equal to the ratio of turbulent shear stress to viscous shear stress, $\tau/(\mu\,\partial U/\partial y)$, but this is a special case.

If direct viscous effects *are* small, L is equal to Ky in the inner layer, K being about 0·4. Clearly, viscous effects are just appreciable at the outer edge of the viscous sublayer, where $u_\tau y/\nu \simeq 30$ if $\partial\tau/\partial y$ is small; therefore, if the critical value of $(\tau/\rho)^{\frac{1}{2}}L/\nu$ is universal, it must be $30K$. Of course this analysis is valid only if L is a unique length scale of the energy-containing eddies, but this is a good first approximation in the inner layer. A more subtle objection is that a purely local criterion cannot be universally valid if a typical eddy wavelength is as large as $0{\cdot}4y$, but, again, local-equilibrium concepts are a good first approximation in the inner layer. The choice of the number here taken as 30, following Preston, depends on the definition of appreciable viscous effects; at $u_\tau y/\nu = 30$, $(\mu\,\partial U/\partial y)/\tau \simeq 0{\cdot}1$.

If we take the start of reverse transition to be a condition in which the viscosity-independent region has shrunk to zero, we see that the criterion for the start of reverse transition is $\{(\tau/\rho)^{\frac{1}{2}}L/\nu\}_{\max} = 30K$: it will appear below that, in practice, the maximum value occurs in, or at the edge of, the inner layer, so that this crude analysis need not be extended to the outer layer where there is no unique length scale. The experimental evidence about the importance of viscous effects further from the surface than the point of maximum eddy Reynolds number is discussed in the appendix.

3. Preston's criterion

Preston, following Landweber, suggested that a radical change in the turbulence structure would start when the logarithmic part of the velocity profile, extending from the outer edge of the sublayer to the inner edge of the outer layer, (say $y/\delta = 0{\cdot}2$) shrank to zero; this occurs when $0{\cdot}2 u_\tau \delta/\nu = 30$ (say). Preston did not consider the case where the shear stress at $y/\delta = 0{\cdot}2$ was appreciably different from the wall shear stress, but his criterion is supported by experimental evidence in pipes and boundary layers.

Now near $y/\delta = 0{\cdot}2$, L departs from its inner layer value of Ky and becomes nearly constant (Bradshaw, Ferriss & Atwell 1967); L or the mixing length are sometimes taken as piecewise-linear functions with a turnover point at $y/\delta = 0{\cdot}2$. If τ decreases *slowly* with increasing y, as in zero pressure gradient, the eddy Reynolds number $(\tau/\rho)^{\frac{1}{2}} L/\nu$ will reach its maximum value near $y/\delta = 0{\cdot}2$. Preston's criterion implies that this maximum value should be $30K$, in agreement with the criterion of § 2; the two derivations are nearer in spirit than in the letter.

4. Patel & Head's criterion

If τ decreases rapidly with increasing y (and assuming $\partial\tau/\partial y$ to be independent of y for convenience), then the maximum value of the eddy Reynolds number $(\tau/\rho)^{\frac{1}{2}} L/\nu$ will be $2K/(3^{\frac{3}{2}}\Delta_\tau)$, at $u_\tau y/\nu = 2/(3\Delta_\tau)$ rather than at $y \doteq 0{\cdot}2\delta$. The maximum value is $30K$ if $\Delta_\tau = -0{\cdot}013$, and $43K$ if $\Delta_\tau = -0{\cdot}009$, the latter being the value of Δ_τ at which Patel & Head found appreciable 'overshoot' in their velocity profiles. In view of the imprecision both of analysis and of experiment, the agreement with the criterion of § 2 is again adequate. It is of course possible that 'overshoot' may be caused by an increase in sublayer thickness (in terms of $u_\tau y/\nu$, $(\tau/\rho)^{\frac{1}{2}} y/\nu$ being constant) *prior* to reverse transition. In a helpful private discussion Dr Head and Dr Patel have pointed out that since, according to their analysis, positive pressure gradient leads to a positive value of their parameter g (which is a rough measure of overshoot) one would expect negative pressure gradient to lead to negative g and thus to an 'undershoot' in the absence of reverse-transition effects. For practical purposes 'overshoot' is an adequate indication of imminent reverse transition in strong favourable pressure gradient.

5. Conclusion

Preston's criterion for the minimum Reynolds number for a turbulent boundary layer in zero pressure gradient, and Patel & Head's criterion for the start of reverse transition in strong favourable pressure gradient, agree well with an eddy Reynolds number criterion, $\{(\tau/\rho)^{\frac{1}{2}} L/\nu\} = 30K$ or $\{(\tau/\rho)^{\frac{1}{2}} y/\nu\}_{\max} = 30$, based on arguments about the overlap of the energy-containing and dissipating ranges of eddy size. The last-mentioned criterion should be generally applicable for engineering purposes; it should, for instance, cover the important case of boundary layers with suction. It is, in effect, a physically-based interpolation between the criteria for zero pressure gradient and for strong favourable pressure gradient. In practice one would take L to be the piecewise-linear function mentioned above or, better, a smooth function as used by Bradshaw *et al.* (1967) and others. The shear stress (in the case of a solid surface) can be taken as roughly $\tau_w + 0{\cdot}5(dp/dx)y$ (Patel & Head 1968) or, more accurately, as

$$\tau_w + \frac{dp}{dx}y + \frac{1}{2\tau_w}\frac{d\tau_w}{dx}\int_0^y U^2 dy,$$

the well-known form obtained by substituting the universal inner-law velocity profile $U = (\tau_w/\rho)^{\frac{1}{2}} f((\tau_w/\rho)^{\frac{1}{2}} y/\nu)$ in the momentum equation. The latter is a convenient form for use with an integral method of boundary layer calculation; more refined methods give τ throughout the layer directly.

Appendix. Conditions near the outer edge of a boundary layer

We have taken reverse transition to start when no part of the boundary layer is free from viscous effects on the shear-producing eddies. This does not preclude the importance of viscous effects in *some* parts of the boundary layer at higher Reynolds numbers. Viscous effects are always important in the viscous sublayer, and in the analogous 'viscous superlayer' (Corrsin & Kistler 1955) whose thickness is of the order of the local Kolmogorov length scale $(\nu^3/\epsilon)^{\frac{1}{4}}$ and which may, like the sublayer, become quite thick at low Reynolds numbers. Indeed, the superlayer may be thicker than the sublayer because ϵ is much less near the superlayer. The resulting reduction of turbulent intensity and shear stress in the outer part of the boundary layer may explain both the gradual disappearance of the 'wake' component of Coles (1962) for $U_1\delta_2/\nu < 5000$ and the gradual spread of the intermittency profile seen in the photographs of Fiedler & Head (1966). Providing that the intermittency factor in the inner layer is still fairly high an eddy Reynolds number based on mean properties should be an adequate criterion for the *start* of reverse transition; thereafter the intermittency factor will fall rapidly. However, the eddy Reynolds number defined in this paper is just a useful correlating parameter for existing data; more work is needed on mixed viscous/turbulent flows, both to explore the nature of the viscous sublayer that provides the inner boundary condition on the turbulent flow and to investigate the direct effect of viscosity in the outer layer at low Reynolds number.

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